Test One

Semester One 2018
UNIT 1 METHODS

Calculator Assumed 40 minutes   **/45 marks**

Scientific Calculator, ClassPad, Formula Sheet and
One page one side of A4 notes is permitted

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Name:	Solutions

Place a tick in the box next to your Mathematics teachers name:

Mr Strain	☐
Ms Sindel	☐
Ms Rimando	☐
Mr Gannon	☐
Mr Young	☐
Mrs Flynn	☐
Ms Ensly	☐

Question 1 $\qquad$ (2, 2, 2 = 6 marks)

Consider the following points, A (4,9) and B (20,12).

    i)   Determine the exact distance from point A to B.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(12-9)^2 + (20-4)^2} \checkmark$$

$$= \sqrt{9+256}$$

$$= \sqrt{265} \approx 16.28 \checkmark$$

    ii)   Determine the midpoint between points A and B.

$$m = \left(\frac{x_2+x_1}{2}, \frac{y_2-y_1}{2}\right)$$

$$= \left(\frac{4+20}{2}, \frac{9+12}{2}\right) \checkmark$$

$$= \left(12, 10\tfrac{1}{2}\right) \checkmark$$

    iii)   If point B was the midpoint of points A and point C. Determine the coordinates of point C.

$$A(4,9) \quad B(20,12) \quad C(x,y)$$

$$(20,12) = \left(\frac{4+x}{2}, \frac{9+y}{2}\right) \checkmark$$

So

$$20 = \frac{4+x}{2} \qquad\qquad 12 = \frac{9+y}{2}$$

$$40 = 4+x \qquad\qquad 24 = 9+y$$

$$x = 36 \qquad\qquad y = 15$$

$$\therefore \text{ Point } C \text{ is } (36,15) \checkmark$$

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Question 2 (2, 2, 1, 3 = 8 marks)

Determine the equation of a line that passes through the point $(-10, 3)$ and :

i) passes through the point $(5, -7)$.

$(-10, 3) \quad (5, -7)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-7-3}{5+10}$

$= \dfrac{-10}{15}$

$= -\dfrac{2}{3}$ ✓

$y = mx + c$

$3 = \dfrac{-2}{3}(-10) + c$

$3 = \dfrac{20}{3} + c$

$c = 3 - \dfrac{20}{3}$

$= -3\dfrac{2}{3}$ ✓

$\therefore \ y = -\dfrac{2}{3}x - 3\dfrac{2}{3}$ ✓

$3y = -2x - 11$

ii) is parallel to the line $y = -5x + 11$.

$m = -5 \quad (-10, 3)$

$y = mx + c$

$3 = -5(-10) + c$ ✓

$3 = 50 + c$ ✓

$c = -47$

$\therefore \ y = -5x - 47$ ✓

iii) is parallel to the y axis.

$x = -10$ ✓

iv) is perpendicular to the line $x - 4y = 9$.

$-4y = -x + 9$

$y = \dfrac{1}{4}x - \dfrac{9}{4}$ ✓

$\therefore \ m_\perp = -4 \quad (-10, 3)$

$y = mx + c$

$3 = -4(-10) + c$

$3 = 40 + c$ ✓

$c = -37$

$\therefore \ y = -4x - 37$ ✓

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Question 3   **(2, 3, 3 = 8 marks)**

Consider the line $5x + my = 21$, where m is a constant.

   i)   In terms of m, determine the y intercept.

$$my = -5x + 21 \checkmark$$
$$y = \frac{-5x}{m} + \frac{21}{m} \checkmark$$

$\therefore$ y intercept is $\left(0, \frac{21}{m}\right)$

   ii)   In terms of m, determine the midpoint of the x and y intercepts.

At $y = 0$ $5x = 21$ $\checkmark$
$$x = \frac{21}{5} \checkmark$$

$\therefore$ x intercept is $\left(\frac{21}{5}, 0\right)$

$$\text{Midpoint} = \left(\frac{0 + \frac{21}{5}}{2}, \frac{\frac{21}{m} + 0}{2}\right) \checkmark$$

$$= \left(\frac{21}{10}, \frac{21}{2m}\right) \checkmark$$

   iii)   Determine the value of m so that the line will never cross $y = 7x$.

$m = 7 \checkmark$

$$5x + my = 21$$
$$my = -5x + 27$$
$$y = \frac{-5x}{m} + \frac{27}{m}.$$

$$\frac{-5}{m} = 7 \checkmark$$

$$m = \frac{-5}{7} \checkmark$$

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Question 4   **(1, 1, 2 = 4 marks)**

Jessica needs to hire a car for a number of days. The hire car company has two options from which she can choose.

       Budget: $15 per day plus $0.25 per km travelled

       Deluxe: $42 per day for unlimited travel

i)   Jessica will hire the car for n days and drive a total of x km.

     a)   Find an expression for the cost, $C, in terms of n for the Deluxe option.

$$C = 42n \quad \checkmark$$

     b)   Find an expression for the cost, $C, in terms of n and x, for the Budget option.

$$C = 15n + 0.25x \quad \checkmark$$

ii)   If Jessica plans to drive a total of 600 km, find the maximum number of days for which she can hire the car so that it is cheaper for her to take the Deluxe option.

If $x = 600$

$$42n < 15n + 0.25(600) \quad \checkmark$$

$$27n < 150$$

$$n < 5.55$$

$\therefore$ 5 days. it is cheaper to take $\checkmark$
the Deluxe option

$$\begin{array}{r} 42 \\ \times 5 \\ \hline 210 \\ 225 \end{array}$$

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Question 5 (2, 2, 2 = 6 marks)

Factorise the following expressions:

i) $4x^2y - 12xy^4$

$$4xy\left(x - 3y^3\right)$$

ii) $x^3 - 3x^2 - 9x + 27$

$$= x^2(x-3) - 9(x-3)$$
$$= (x^2 - 9)(x-3)$$
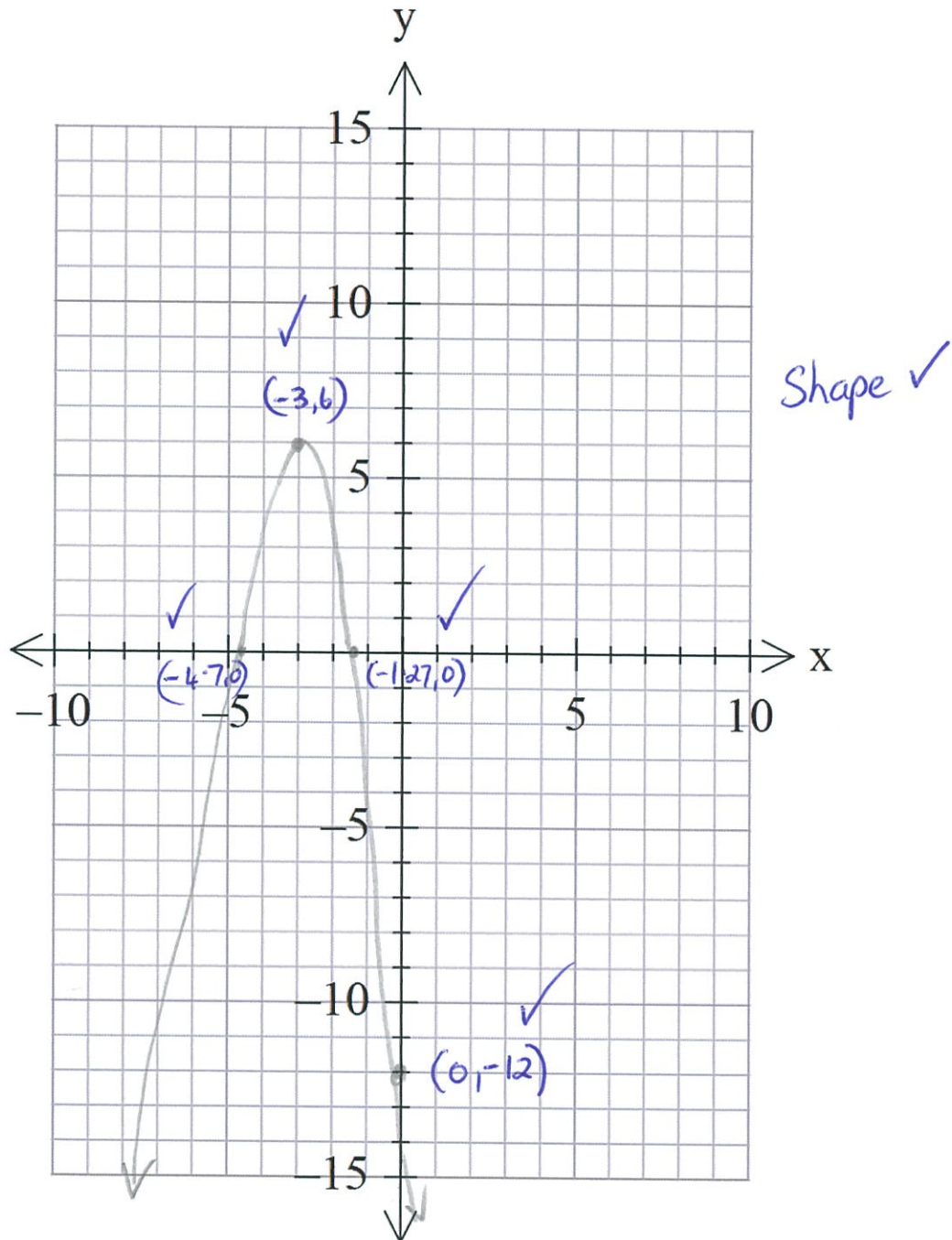$$= (x-3)(x+3)(x-3) = (x-3)^2(x+3)$$

iii) $18x^2 + 33x - 30$

$$= 3(3x - 2)(2x + 5)$$

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Question 6

On the axes below, sketch the parabola $y = -2(x + 3)^2 + 6$ showing all major features such as line of symmetry, turning points, x intercepts (if any).

Shape ✓

(-3,6)

(-4·7,0)

(-1·27,0)

(0,-12)

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Question 7 (2, 2 =4 marks)

For each of the following write down the equation of a parabola that satisfies the following:

(No need to simplify)

i) A quadratic with intercepts $(4,0)$ and $(-7,0)$ with a y intercept of $(0, -56)$.

$$y = a(x-4)(x+7) \quad \checkmark$$

At $(0, -56)$

$$-56 = a(-4)(7)$$
$$= -28a$$
$$a = 2$$

$$\therefore \quad y = 2(x-4)(x+7) \quad \checkmark$$

ii) A quadratic with a maximum turning point $(7,1)$ and an x intercept $(10,0)$.

TP $(7,1)$

$$y = a(x-7)^2 + 1 \quad \checkmark$$

At $(10,0)$
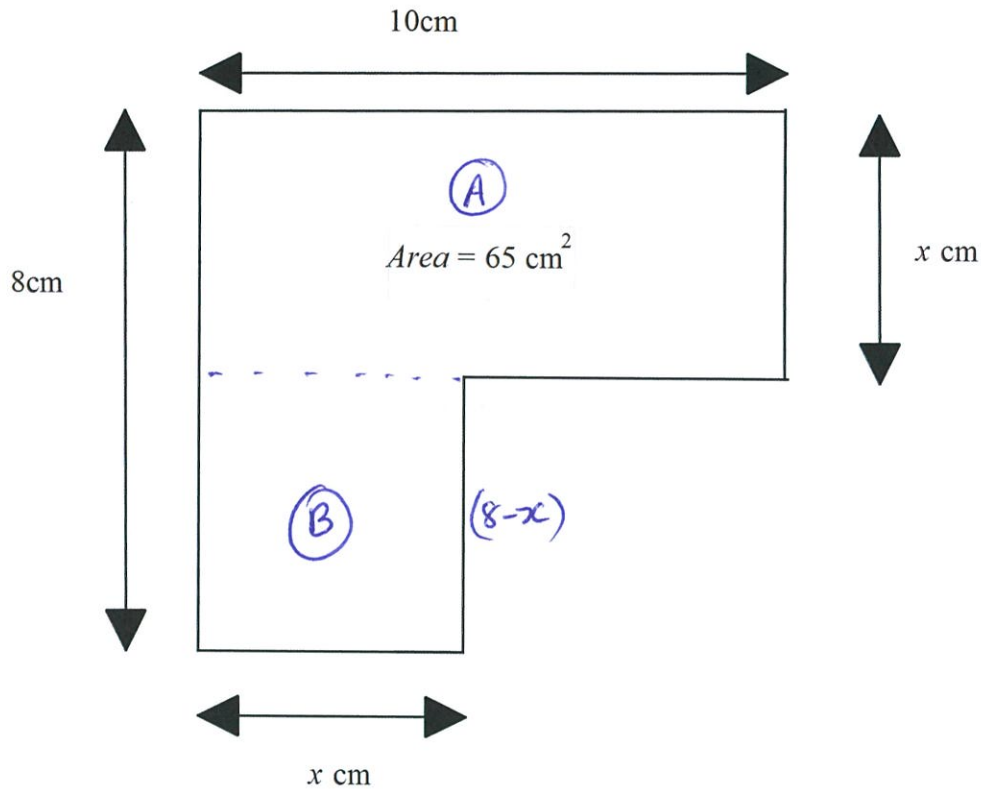
$$0 = a(3)^2 + 1$$
$$= 9a + 1$$
$$-1 = 9a$$
$$a = -\tfrac{1}{9}$$

$$\therefore \quad y = -\tfrac{1}{9}(x-7)^2 + 1 \quad \checkmark$$

Question 8 (4 marks)

Determine the value of x for the shape below.

10cm

(A)

$Area = 65$ cm^2

8cm

x cm

(B)

$(8-x)$

x cm

Area (A) $= 10x$

Area (B) $= (8-x)x$

$= -x^2 + 8x$ ✓

Total Area

$65 = -x^2 + 8x + 10x$ ✓

$0 = -x^2 + 18x - 65$ ✓

$= x^2 - 18x + 65$

$= (x-5)(x-13)$ ✓

∴ $x = 5$ or 13

As $13 > 10$, 13 is not a solution ✓

∴ $x = 5$ cm

End of test

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